# METHOD AND SYSTEM FOR MODULATING AND DETECTING HIGH DATARATE SYMBOL COMMUNICATIONS

## BACKGROUND OF THE INVENTION

### 1. Field of the Invention

The present invention relates generally to communications systems, and more specifically, to a digital satellite communications modulation and detection scheme for multiple-carrier link operation.

### 2. Background of the Invention

Digital communication systems are presently prevalent in both voice and data communications equipment. One such application is in satellite data link communications equipment, where minimum spectral efficiencies are required for uplink/downlink equipment designed to meet specific standards. In order to meet high datarate requirements on a band-limited channel such as a satellite connection, a minimum spectral efficiency must be met in conjunction with a low symbol error probability requirement. The symbol error probability is dictated by the energy per transmitted bit versus the noise spectral density ($E_{bt}/N_o$). System requirements for gain and noise level that determine $E_{bt}/N_o$ are dictated by maintaining minimum energy and maximum noise levels at each link in the communications chain.

The modulation schemes employed in the above-described

communications systems have typically been quadrature amplitude modulation (QAM) or quadrature phase-shift keying (QPSK), providing a band-limited signal with a high spectral efficiency. Each phase state (for QPSK) or amplitude-phase state (for QAM) is mapped to a particular value for an interval in the signal (often referred to as a "chip"). The particular phase/amplitude states used to represent information collectively form a "constellation", which is so-called because of the shape defined by a phase-amplitude diagram of the particular modulation states. Selection of the constellation and mapping of multiple chip sequences to symbols provides for rejection of disallowed combinations or sequences, lowering the symbol error probability of the link by increasing the effective $E_{bt}/N_o$.

For satellite systems that operate the output amplifiers near saturation, as in a single-carrier system, phase-shift-keying (PSK) has traditionally been a modulation of choice, as PSK detection is not degraded for amplitude compression of the signals due to non-linearity introduced in a high power amplifier (HPA). However, in multi-band satellite links, where multiple carriers carry multiple data streams, non-linear operation of the HPA is avoided, as signal amplitude distortion and inter-channel interference is introduced by any non-linearity in the link.

Selection of a symbol rate with respect to a known bandwidth generally dictates a spectral efficiency and traditionally a particular modulation type. However, for PSK, as the number of states per chip are increased by increasing modulation order, the ability to distinguish between states decreases and the ability to determine a unique carrier phase reference without cycle slip deteriorates. A transition from QPSK, which provides a maximum spectral efficiency of 2 bits/s/Hz to a system in which a 3 bit/s/Hz requirement may be met, requires (for PSK) an 8 state system known as 8PSK.

Therefore, it would be desirable to provide a communications link providing a spectral efficiency of 3 bits/s/Hz that outperforms 8PSK so that improved link performance is obtained. It would further be desirable to improve carrier detection performance for a selected constellation.

## SUMMARY OF THE INVENTION

The above stated objectives are achieved in a method and communications system for modulating and detecting data on a satellite communications channel. A novel combination of quadrature amplitude modulation using a predetermined constellation, and an optimized log-likelihood mapping scheme for detecting points of the predetermined constellation provides optimized bit error rate BER performance at 3 bits/s/Hz.

The constellation has point values with two points of differing amplitude and having a zero reference phase, and two other points having corresponding amplitude to the first two points with a 180 degree reference phase. The remaining four points are located symmetrically above and below the lower amplitude points of the zero reference phase and 180 degree reference phase set. The points may have complex locations substantially equal to (K/2,0), (3K/2,0), (K/2,K), (-K/2,K), (-K/2,0), (-3K/2,0), (-K/2,-K) and (K/2,K), where K is an arbitrary amplitude reference and further including all phase rotations of the constellation.

The mapping is performed in groupings, with three groupings one associated with each bit of a 3-bit symbol. Each grouping includes four points in a subgroup associated with a logical "0" for the associated bit and four points in another subgroup

associated with a logical "1" and each grouping has two points in common with the other groupings. A unique point is thus selected by the intersection of the subgroups determined by an input 3-bit symbol value.

Finally, a log-likelihood metric is applied to the detector mapping, so that subgroup membership is determined for each grouping (symbol bit) in a non-linear fashion. The log-likelihood outputs are provided to a codec, which may be a turbo product decoder, or low density parity check decoder providing forward error correction.

A novel carrier phase detector for extracting a coherent demodulation reference may be used within the above-described system to further enhance system performance. The carrier phase detector uses amplitude information of received constellation points to selectively include only zero and 180 degree phase reference received points in the carrier phase detector output.

The foregoing and other objectives, features, and advantages of the invention will be apparent from the following, more particular, description of the preferred embodiment of the invention, as illustrated in the accompanying drawings.

**BRIEF DESCRIPTION OF THE DRAWINGS**

**Figure 1** is a block diagram depicting a satellite network communications system in accordance with an embodiment of the present invention.

**Figure 2** is a block diagram of a transmitter in accordance with an embodiment of the present invention.

**Figure 3** is a block diagram of a receiver/demodulator in accordance with an embodiment of the present invention.

**Figure 4** is a block diagram of a carrier phase detector in accordance with an embodiment of the present invention.

**Figures 5A** and **5B** are constellation diagrams for illustrating the constellation employed in the present invention.

**Figures 6A-6D** are constellation diagrams for illustrating the mappings used in the present invention.

**Figures 7A-7C** are contour plots depicting log likelihood metrics used in the present invention.

**Figure 8** is a graph depicting BER performance of a system in accordance with an embodiment of the present invention.

## DESCRIPTION OF ILLUSTRATIVE EMBODIMENT

The present invention includes methods and systems for communicating digital data at fixed spectral efficiencies, in particular a standardized spectral efficiency of 3 bits/second/Hz as well as a receiver apparatus embodying the methods of the present invention. Generally, the fixed spectral efficiency channels alluded to above are dictated in satellite communications channels and the techniques of the present invention are particularly applicable to digital satellite communications. However, the techniques embodied herein may be applied to other forms of communication and the present invention should be understood to the use of the techniques described herein as applied to other communications systems.

Highly efficient forward error correction (FEC) techniques have been developed and incorporated into communications systems that decrease the output symbol error rate by correcting errors and therefore effectively increasing $E_{bt}/N_o$. High-speed FEC codes known as Turbo Product Codes (TPCs) and Low Density Parity Check (LDPCs) codes are employed in satellite communication systems (as well as in other communication systems) that provide low BER with low computational overhead. In the past, the efficiency of a data communications channel employing FECs was largely dictated by the FEC scheme employed, but with highly efficient FEC codes,

the modulation scheme becomes a determining factor in the overall spectral efficiency. The present invention provides a modulation and mapping scheme, as well as a detector that advantageously uses the features of the modulation and mapping scheme to improve the performance of a communications channel..

Referring now to the figures and in particular to **Figure 1**, a satellite network communications channel within which the present invention may be embodied is depicted in a block diagram. A first uplink/downlink includes a router **11,** a transmitter-receiver **12** and an antenna **13**. Router **11** is coupled to a network **10** that exchanges packets with the first uplink/downlink. In the context of the present invention, packets indicate time-division multiplex (TDM) data as well as packets for data communications such as Internet protocol (IP) packets. The present invention is not dependent on a particular data format and may be applied to continuous data streams as well. A satellite **15** contains a transmitter-receiver, transponder or other suitable circuitry for receiving and transmitting information using an antenna **14**. The configuration of a separate transmitter-receiver **12** and router **11** is an illustrative configuration. Packets transmitted via first uplink/downlink to satellite **15** may be forwarded to a second uplink/downlink that includes a router **18,** a transmitter-receiver **17** and an antenna **16**. Router **18** is coupled to a network **19** which can interchange packets with the second uplink/downlink. Other

configurations such as other types of non-network digital communications systems will include data connection blocks other than routers **11**, **18** and network connections **10**, **19**, such as data multiplexers for digital telephony or digital switches with analog front-ends for analog telephone switch satellite connections. Other digital communication configurations such as radio are contemplated by the present invention and represent additional embodiments thereof.

While the illustrative embodiment is directed to communications channels involving a satellite uplink/downlink it will be understood by those of ordinary skill in the art that the present invention may be used with other communications channels, and that the advantages of the present invention are particularly applicable to those communications channels that have fixed spectral efficiency requirements dictated by fixed channel bandwidth and bit error rate (BER).

The present invention provides a spectral efficiency of 3 bits/sec/Hz at lowered power levels commensurate with new coding techniques such as Turbo Product codes and Low Density Parity Check Codes. A need for a 3 bit per symbol (8 point) modulation scheme that is more robust than conventional 8PSK at low $E_{bt}/N0$ has been identified, since error correction coding has improved to the point that it is now necessary not only to look at the bit

error rate versus $E_{bt}/N0$ of a system, it is also necessary to find a scheme which can be robustly demodulated at the $E_{bt}/N0$ target for a selected forward error correction (FEC) scheme.

Referring now to **Figure 2**, a block diagram of a transmission/modulation system in accordance with an embodiment of the present invention is depicted. Data is provided to an FEC circuit, in the present embodiment a TPC encoder **22**, which is generally an encoder integrated circuit or programmable array having an FEC circuit embodied therein. Turbo product codes are well known in the art as FEC codes that provide improvements over more traditional one-dimensional coding. In accordance with alternative embodiments of the present invention, the various blocks providing for encoding and decoding of turbo product codes may be replaced with low density parity check (LDPC) encoders and decoders that provide for use of LDPC coding. In general, it is not the particular type of FEC coding that the present invention concerns as much as it is the fact that an FEC corrected channel as available today dictates a maximum BER that must be maintained over the range of $Eb_t/NO$ that may be encountered in the system and that the modulation scheme can provide improved BER for wider ranges of $Eb_t/NO$ than may be provided by 8PSK. Therefore, other types of FEC coding that generate such requirement may find benefit from the application of the constellation and mapping techniques of the present invention, as well as the detection

metrics and carrier phase detector employed in the disclosed embodiments of the present invention.

TPC is a technique that uses conventional block codes such as Hamming or Parity codes in multiple dimensions, and uses iterative decoding with a Soft Input Soft Output or SISO decoding engine to achieve excellent coding gain. U.S. Patent No. 6,526,538 discloses TPC techniques and is incorporated herein by reference.

TPC coding techniques are particularly advantageous for code rates N/K approaching 1 (N represents the data length of a block and K is the symbol or code length), where not much additional parity is transmitted. The use of Turbo Product Coding in the present invention provides excellent performance in a communications system, replacing a more conventional modulation scheme in order to achieve greater spectral efficiency and a lower transmission power requirement for a given error rate. LDPC codes provide similar advantages.

The FEC encoded output of turbo product encoder **22** is provided to a mapper **23**, that is generally a memory-based lookup table providing mappings from 3-bit code outputs of turbo product encoder to detection probabilities for points on a QAM constellation implemented by QAM constellation signal generator

**24**. The constellation provided by QAM constellation signal generator **24** is a particular constellation and rotations thereof that produce an optimum result in combination with the mapping techniques of the present invention. The mappings provided by mapper **23** and the constellation employed in QAM generator **24** will be discussed in detail below with respect to Figures 5 and 6. The output of QAM generator is provided to a mixer **25** that upconverts the QAM generator output via local oscillator (LO) **26** and the output of mixer **25** is provided to high power amplifier (HPA) **28** for producing a signal provided to an antenna, such as antennas **16** and **13** of **Figure 1**. In particular, the constellation used in the present invention provides detection advantages for signals processed in an HPA that is operated in the linear region, such as in multi-channel satellite communication systems, where power may only be allocated only for active channels and where inter-channel interference requirements dictate that the HPA is operated below non-linear regions nearing saturation of the HPA.

Referring now to **Figure 3**, a block diagram of a receiver/demodulator in accordance with an embodiment of the present invention is shown below. An input signal comprising bit frames having modulation phase and amplitude set by the constellation points is provided to a complex multiplier **31**. Complex multiplier **31** demodulates the input signal in conformity with the output of an oscillator **35** having a phase and amplitude

locked by a phase-lock loop implemented by a carrier phase detector **33** a loop filter **34** in combination with the oscillator and signal path to phase detector **33**.

The output of complex multiplier **32** is coupled to a set of matched filters **32** that extract quadrature (I,Q) channel outputs from the demodulated output signal of mixer **31** and provide them to carrier phase detector **33** and a look-up table **37**. In practice, matched filters **32** are digital implementations and include either A/D converters for accepting an output of mixer **31** or mixer **31** itself is a digital complex multiplier operating on a digital representation of a sampled input signal. Look-up table performs a detection operation on the (I,Q) signals, providing three bit detection probability outputs to turbo product decoder **38**, which provides the final decoded output data. Look-up table **37** provides novel functions within the receiver of the present invention, converting the (I,Q) inputs to a series of coefficients for providing codec inputs to TPC decoder **38**. The coefficients are determined by a log-likelihood mapping of equal probability curves associating I and Q amplitude values with three groupings of points of the QAM constellation employed in modulation scheme of the present invention.

Prior demodulation schemes typically have not employed decoding of the above-described type, as PSK modulation schemes

do not typically benefit from such mappings and prior QAM schemes are not advantaged sufficiently by a non-linear detection mechanism so as to justify the cost of look-up table **37**. For example, the depicted embodiment may employ a look-up table having 16 probability levels (4 bits) for each of the 3 grouping decodes. The I,Q input values may be 8 bits each, requiring a 64Kbyte 12-bit memory that has an access time appropriate to the data rate of the system, which is generally prohibitive. Folding techniques may be used to reduce the memory size required by taking advantages of symmetry in the mapping functions, but generally a simplified mapping output is provided rather than a log-likelihood mapping contour as used in the present invention. The particular constellation and mappings used in the present invention provide an advantage when mapped by the log-likelihood look-up table **37**, so that the cost of the above-described memory is justified by an increase in $E_{bt}/N0$ on the order of 0.4dB.

The present invention also uses the constellation used in the method and systems of the present invention advantageously by incorporating a carrier detector in accordance with another embodiment of the present invention. Referring now to **Figure 4**, a carrier phase detector **40** in accordance with an embodiment of the invention is shown that may be used as carrier phase detector **33** of **Figure 3**. I and Q channel inputs from matched filters **32** are provided to squaring functions **42A** and **42B** and outputs of

squaring functions **42A** and **42B** are summed by summation **43** to provide a measure of the complex amplitude of the received signal (signal radius). The input signal radius is detected by a pair of comparators **44A** and **44B** having outputs combined by OR gate **45**. The OR gate **45** output enables an output of carrier phase detector **40** so that only when the signal radius exceeds the window established by the low and high threshold reference inputs of comparators **44A** and **44B**, is the phase-lock detector output applied to an attached loop filter. Computing the radius exactly would require a "square root" operation, which is a complex calculation. It is relatively less complex to square the I and Q inputs and add them. The resulting sum is equal to the square of the radius, but by adjusting the High and Low amplitude detection threshold values to compensate yields the desired result of classifying the constellation points into amplitude sets.

The low threshold is set between the level corresponding to the radius of the inner 2 points (normalized as radius 1) and the level of the radius of the 4 points with normalized radius of 2.24. This gives a normalized threshold value of about $1.6^2$. The high threshold is set between the level of the 4 points with normalized radius of 2.24 and the outmost 2 points with normalized radius of 3, with a value of about $2.6^2$. The level comparators and the logical OR gate generate an enable signal to turn the output of the detector on or off. The detector is on if

the square of the radius is lower than the low threshold or higher than the high threshold.

The low and high threshold reference inputs of comparators **44A** and **44B** are set so that the lowest amplitude and highest amplitude points contribute to (and are thereby correlated by) phase detector **40** output, and a property of the constellation employed in the present invention as described below is that the lowest and highest amplitude points have the same or 180 degree phase relationship to each other, whereas the points having amplitudes within the window effectively excluded by comparators **44A** and **44B** are points having a differing phase. The sign of the phase of the points admitted to the phase detector is corrected by a multiplier **47** that multiplies the Q channel input to phase detector **40** by a sign derived from the I channel input via a sign detector (comparator) **41**. The use of the Q channel provides a lock with respect to the reference phase aligned with the I channel, as is well known in the art. As the phase of the received signal deviates from the I channel aligned phase, the amplitude of the Q channel signal increases, thus increasing the level applied to phase detector **40** output for signals above or below the window of exclusion provided by comparators **44A** and **44B**. The sign of the I channel controlling the application of Q or -Q to the output of phase detector **40** provides the proper

polarity so that the loop filter will pull-in the oscillator and lock the phase-lock loop.

Since the snowflake constellation is rotationally symmetric over π radians as opposed to, for example, π/4 radians for 8 PSK (8 point phase shift-key modulation), the achievable lock stability is greater. The radial amplitude is independent of rotation of the constellation, so the set to which a given constellation point belongs can be determined even when the constellation is rotating (before the carrier loop is locked). The 2 inner and outer points fall on the x-axis when the constellation is locked, and they will therefore be correctly stabilized by a π symmetric detector as described above. An alternative to the above-described phase detector is an I*Q detector, but the described detector that computes Q*sign (I), has greater linear phase range at high Ebt/N0 and can be implemented using with a simplified inverse circuit rather than full hardware multiply.

At the target operating point, the carrier phase detector of the present invention retains roughly ½ of its original gain while an 8PSK detector loses 90% of its original gain. The result is performance improvements with respect to cycle skipping. Cycle skip performance is a non-linear phenomenon and

is very difficult to quantify. A rough approximation to the difference between using an 8PSK detector and a BPSK detector is that the order of the non-linearity necessary to operate an 8PSK detector is 8 vs. 2. There is approximately a 3dB signal to noise penalty for each factor of 2 of non-linearity that must be introduced. Therefore, the selected snowflake constellation, when received using the phase detector of the present invention, can be tracked roughly 6 dB below the point where an 8PSK detector will fail. The phase detector described above substantially extends the range of utility for modulation at 3 bits/sec/Hz.

Referring now to **Figures 5A** and **5B**, the constellation employed in the present invention is contrasted with an 8PSK constellation, by way of illustration. **Figure 5A** depicts an 8PSK constellation resembling a circle, where the radial position of the points shown in the diagram represents the amplitude of a signal frame and the angular position represents the phase. All points have equal amplitude and the phase of the points are distributed at 45 degree increments around the circle. **Figure 5B** depicts the QAM constellation employed in embodiments of the present invention, resembling a snowflake. Four points are located along the zero/180 degree phase axis and represent the points permitted to affect carrier phase detector **40** outputs as described above. Note that the amplitude of the non-zero phase

points is in between the amplitudes of the inner points and outer points of the zero/180 degree points. The non-zero phase points are excluded from the phase detector **40** output by comparators **42A** and **42B**. Thus, the carrier phase detector of the present invention uses the amplitude information embedded in the constellation of **Figure 5A** to improve carrier tracking. The constellation provides further improvement in detection in combination with a mapping of points as will be described below.

The Snowflake constellation may be described as 8 points on an equally spaced rectangular grid such that 4 points are in a horizontal line. To these are added 2 pairs of additional points equally spaced above and below the center 2 points in the horizontal line. Although we define this constellation (and subsequent mappings, metrics and phase detector) in terms of horizontal and vertical spacing, the performance is independent of any fixed rotation, and therefore rotations of the constellation represent equivalent modulation schemes. The depicted rotation includes the points having complex locations substantially equal to (K/2,0), (3K/2,0), (K/2,K), (-K/2,K), (-K/2,0), (-3K/2,0), (-K/2,-K) and (K/2,K), where K is an arbitrary amplitude reference representing the spacing between the points on the I axis (horizontal axis) of the constellation diagram. Variations from the set of points described above will tend to reduce performance, but there may be advantages in

reducing the amplitudes of the (+/-3K/2,0) points to reduce the peak power level, and it should be understood that minor variations in the phase or amplitude of the points are equivalent values contemplated by the present invention.

Referring now to **Figures 6A-6D**, a mapping for the constellation shown in **Figure 5B** is illustrated. Mapping is the process of coding the 3 bits represented by the constellation signal frames into 1 of the 8 constellation points. There 8! or 40,320 possible combinations of the 8 points. Gray coding where possible is generally a preferred starting point for determining a mapping. The mapping is selected to minimize the total possible number of bit errors as a function of symbol errors. Within the 40,320 possible mappings mentioned above, 48 have optimal BER properties. The optimum mappings are determined by analyzing the bit error possibilities for a given symbol error rate, when coding 3 bits into 1 of 8 points, as will be described below.

The mapping of the present invention as embodied in look-up table **37** of **Figure 3** and mapper **23** of **Figure 2**, is performed by dividing the eight points into three groupings, one associated with each encoded bit. Each grouping has two subgroups of four points, one subgroup associated with a logical "1" value for the associated bit, and another subgroup associated with a logical

"0" value for the associated bit. Each of **Figures 6A-6C** represent one of the groupings, and **Figure 6D** shows a map of all of the groupings and all dividing lines distinguishing the groupings. Grouping 1 depicted in **Figure 6A** maps a first logical value for the associated bit to the points on the left half-plane of the diagram and the other logical value to the points on the right half-plane. Grouping 2 depicted in **Figure 6B** maps the four lower center points to a first logical value and the other points to the other logical value. Grouping 3 depicted in **Figure 6C** maps the four upper center points to a first logical value and the other points to the other logical value. Thus three bits are represented by the constellation and a unique combination of the values (a 3-bit word) is determined for each constellation point by virtue of the point's membership in a particular subgroup of each grouping.

Optimum performance for a recursive coding scheme is achieved by using log-likelihood metrics for decoding. However, as mentioned above, linear metrics have been used in the past, as linear metrics are simple to generate and the memory required for implementing log-likelihood metrics is prohibitive. Referring now to **Figures 7A-7C**, contour plots of log-likelihood metrics for the groupings described above are shown. The plots show optimum receive slicing for Turbo Product Codes with the slicing levels on a per bit basis from the following equation:

$$LLR_{idx} := \ln\left[\frac{\displaystyle\sum_{j=0}^{7} \mathrm{if}\left[Code_j = 1, e^{-\left[\frac{\left(Idot_j - Itest_{idx}\right)^2 + \left(Qdot_j - Qtest_{idx}\right)^2}{2\cdot\sigma^2}\right]}, 0\right]}{\displaystyle\sum_{j=0}^{7} \mathrm{if}\left[Code_j = 0, e^{-\left[\frac{\left(Idot_j - Itest_{idx}\right)^2 + \left(Qdot_j - Qtest_{idx}\right)^2}{2\cdot\sigma^2}\right]}, 0\right]}\right]$$

The contours shown in **Figure 7A** correspond to the log-likelihood equal probability contours for grouping I of **Figure 6A**. The contours shown in **Figure 7B** correspond to the log-likelihood equal probability contours for grouping II of **Figure 6B**. The contours shown in **Figure 7C** correspond to the log-likelihood equal probability contours for grouping III of **Figure 6C**. Worthy of note is that while approximations using a linear

metric for Grouping I would generally be close enough not to degrade the optimum BER of the system of the present invention, linear metrics used for Grouping II and Grouping III would deviate significantly from the ideal contours of **Figures 7B** and **7C**. Therefore, use of the log-likelihood contours included in the method and system of the present invention provides a distinct improvement in conjunction with the constellation and groupings used in the present invention. An implementation of a receiver including the contours of Figures **7A-7C** may be made by coding these contours into a memory implementing look-up table **37** and addressed by demodulator I and Q outputs to produce a numerical value in conformity with the contour, which is then delivered to the input of the error correction decoder.

In order to evaluate the efficiency of a constellation it is necessary to estimate the constellation's error performance. This is done by observing the effect of adding Gaussian noise to the In Phase (I) and Quadrature (Q) channels of each constellation (horizontal and vertical axis respectively on the plots). Given a regular constellation, the probability of a symbol error (transmitting a given point from the 8 possibilities and receiving a different one due to additive noise) can be computed from the constellation geometry and the Normal Probability distribution. A mathematical variant of the Normal distribution is used, known as the "Q" function. The Q function

gives the single sided probability in the "tail" of the Normal probability curve as a function of a given standard deviation. The Symbol error probability curve is determined by computing the equivalent standard deviation for an 8-point constellation, modified by a Geometry Coefficient which is calculated based on the Euclidian distance from a point to the error threshold. The equivalent standard deviation is calculated as a function of the Energy per Transmitted bit to Noise spectral density, or $Eb_tN0$. The Geometry Coefficient modifies the standard deviation. A larger value of Geometry Coefficient is better, with slight variations being significant as the result is used as an exponent. The equations are as follows:

$$\text{Q function( X)} = \frac{1}{\sqrt{2*\pi}} * \int_{x}^{\infty} e^{-\frac{y^2}{2}} *dy$$

$$\text{Symbol Error Probability} = Q(\sqrt{2*3*EbtN0} * Geometry\_Coeff)$$

It is then necessary to compute the bit error probability from the symbol error probability. The bit error probability may be approximated by investigating different "mappings".

For a selected mapping, an average number of bit errors per symbol error can be evaluated, known as a "Mapping Coefficient". The selected snowflake constellation yields a mapping coefficient of 0.9167 and a geometry coefficient of 0.4472, where a typical 8

PSK mapping yields a mapping coefficient of 0.6667 and a geometry coefficient of 0.3827.

The equation for estimated BER is

$$\text{Estimated BER} = \mathit{Mapping\_Coeff} * Q(\sqrt{2*3*EbtN0}*Geometry\_Coeff)$$

The above equation provides a lower bound on the BER, but the actual BER is generally within .05 dB of the estimate. "Monte Carlo" simulations were run to confirm the results of the equations. The BER estimate curves for the snowflake constellation using the above-described optimal mapping versus an 8PSK BER estimate are shown in Figure 8. The snowflake constellation (dotted line) is visibly better than 8PSK (solid line) over a wide range of EbtN0. The results are valid for channels that are average power limited. Some satellite channels are run with the amplifier near saturation and 8PSK may be advantageous in those applications. Much of the satellite market requires a transmission having multiple carriers sharing a transponder and the results presented above are valid for linear amplifier (transponder HPA) operation. Therefore the snowflake constellation provides superior performance in the above described applications.

The combination of the snowflake constellation, mapping for the constellation, the improved carrier phase detector, log-likelihood metrics and TPC encoding and decoding provide a highly spectrum and power efficient data communications link.

While the invention has been particularly shown and described with reference to the preferred embodiments thereof, it will be understood by those skilled in the art that the foregoing and other changes in form, and details may be made therein without departing from the spirit and scope of the invention.